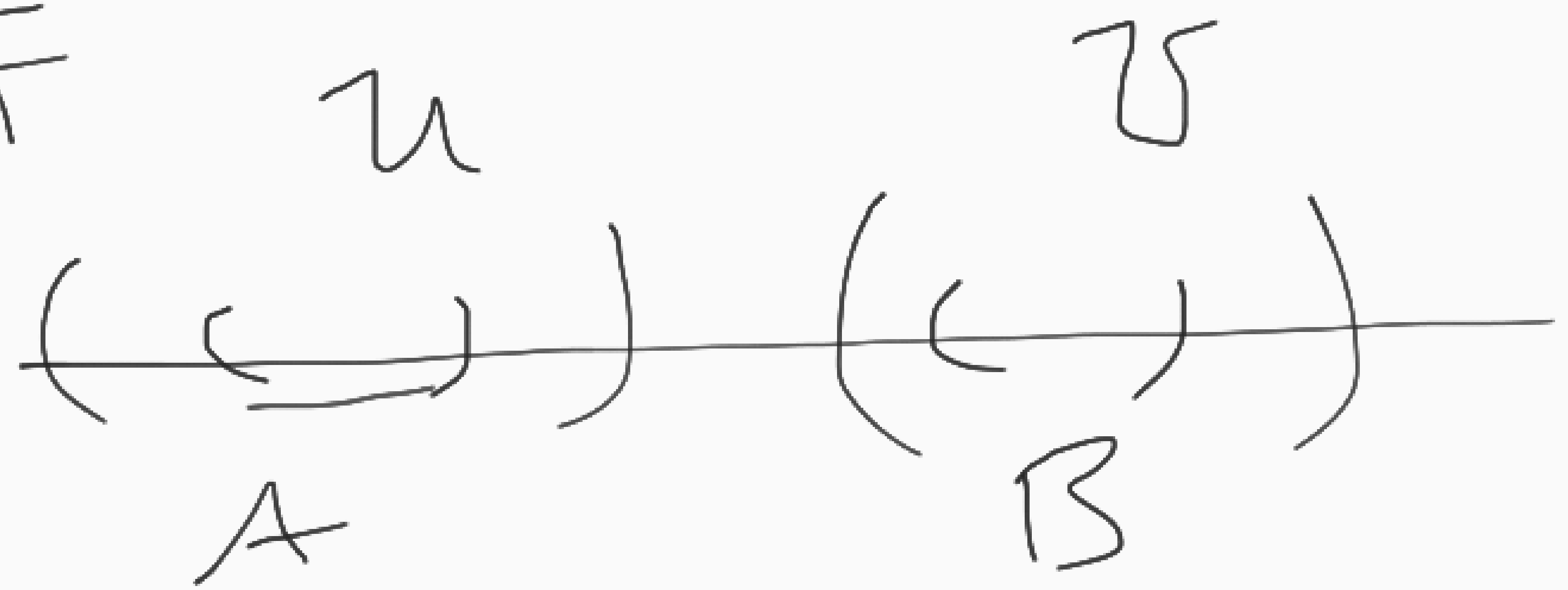
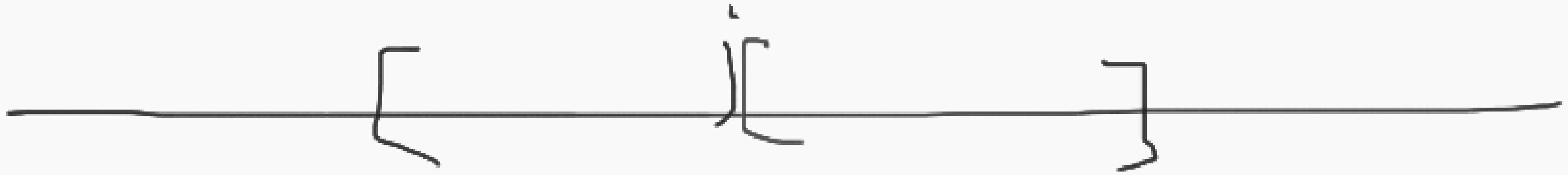
2 a) F





b) $\partial A = \bar{A} \setminus A^{\circ} = A \setminus A = \phi$ T.

$E \subseteq X$, $A \subseteq E$ IS OPEN IN E

$\iff A = E \cap U$

$\exists\, U$ OPEN IN X

c) F $A = [0,1)$

d) F $\rho(1,-1) = 0$ BUT $1 \neq -1$.

$$\rho(x,y) = |x| - |y|$$

③ PF: LET $\{x_n\}$ BE CAUCHY.

$\exists N$ S.T. $n, m \geq N \Rightarrow \rho(x_n, x_m) < 1$

$x_n = x_m$

$\exists N$ S.T. $n \geq N \Rightarrow x_n = x_N$

SHOW $x_n \to x_N$.

LET $\varepsilon > 0$. LET $n \geq N$.

THEN $\rho(x_n, x_N) = \rho(x_N, x_N) = 0 < \varepsilon$.

4 $a \in X$. $f : X \to \mathbb{R}$

$x \mapsto \rho(x, a)$

SHOW f IS CONT AT EACH $y \in X$.

LET $y \in X$, $\varepsilon > 0$. LET $\rho(x, y) < \delta$

SHOW $|f(x) - f(y)| < \varepsilon$

I.E. $|\rho(x,a) - \rho(y,a)| < \varepsilon$

$$\rho(x,a) \leq \rho(x,y) + \rho(y,a)$$

$$\Rightarrow \rho(x,a) - \rho(y,a) \leq \rho(x,y)$$

AND $\rho(y,a) - \rho(x,a) \leq \rho(x,y)$

$$\Rightarrow |\rho(x,a) - \rho(y,a)| \leq \rho(x,y)$$

LET $\delta = \varepsilon$. $\rho(x,y) < \delta$

$$\Rightarrow |f(x) - f(y)| \leq \rho(x,y) < \delta = \varepsilon.$$

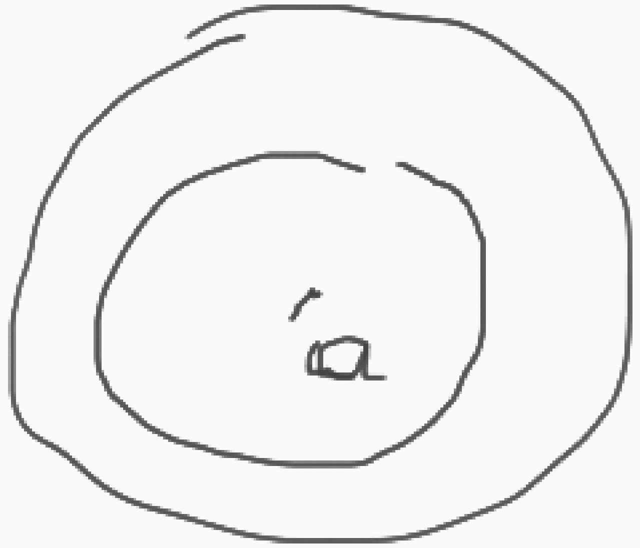
LET $I = (c, d)$

$$f^{-1}((c, d)) = \{x \in X : c < f(x) < d\}$$

$$= \{x \in X : c < \rho(x, a) < d\}$$

$$= B_d(a) \cap \overline{B_c(a)}^{\,c}$$

IS OPEN.



LET U BE OPEN IN $\mathbb{R}$.

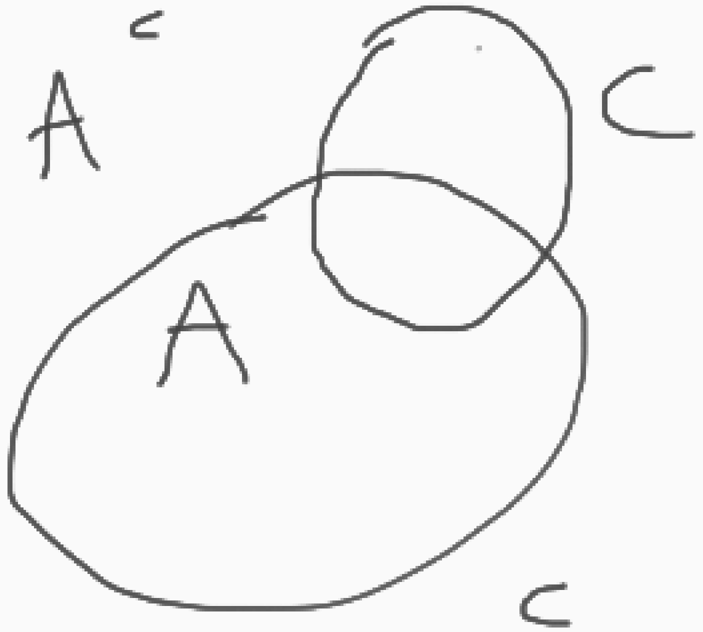
$$U = \bigcup (\, , \,) \Rightarrow f^{-1}(U)$$

$$= f^{-1}\left(\bigcup (\ ,\ ) \right) = \bigcup f^{-1}((\ ,\ ))$$

$$= \bigcup \text{OPEN} \quad = \text{OPEN}.$$

(6) a) $\partial A = \overline{A} \setminus A^{\circ} \Rightarrow \overline{A} = A^{\circ} \cup \partial A$

b)



ASSUME $C \cap \partial A = \emptyset$

$$X = \overline{A} \cup \overline{A}^{c}$$

$$C \subseteq \overline{A} \cup \overline{A}^{c} \subseteq \partial A \cup A^{\circ} \cup \overline{A}^{c}$$

$$\Rightarrow C \subseteq A^{\circ} \cup \overline{A}^{c} \Rightarrow C \subseteq A^{\circ} \text{ OR } C \subseteq \overline{A}^{c}$$

$C \subseteq A^{\circ} \subseteq A \Rightarrow C \cap A^{c} = \phi \Rightarrow \Leftarrow$

$C \subseteq \bar{A}^{c} \subseteq A^{c} \Rightarrow C \cap A = \phi \Rightarrow \Leftarrow$,

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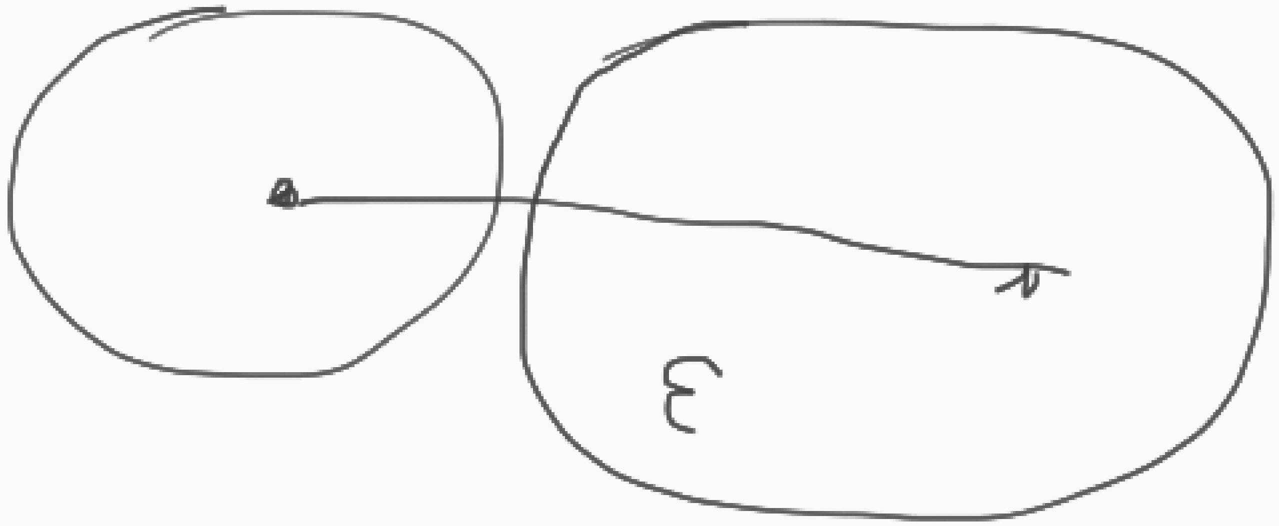
2 a) F $\quad A = (0,1)$, $B = (1,2)$

c) F $\quad \phi \quad \phi$

d) F $\quad A = (0,1)$, $B = \{0\} \Rightarrow AB = \{0\}$

④ $A \subseteq X$. $\exists\, \varepsilon > 0$ s.t. $\rho(a,b) \geq \varepsilon$ $\forall a \neq b$ in A.

A compact $\iff$ A is finite



Pf: $(\Longleftarrow)$ ✓

$\{B_{\frac{\varepsilon}{2}}(a)\}_{a \in A}$ open covering of A.

$\exists \{a_1, \dots, a_n\} \subseteq A$ s.t.

$$A \subseteq \bigcup_{j=1}^{n} B_{\frac{\varepsilon}{2}}(a_j)$$

3 SHOW $\rho(x,y) \le \rho(x,z) + \rho(z,y)$

$\rho(x,y)$

$$x = (x_1, x_2)$$
$$y = (y_1, y_2)$$
$$z = (z_1, z_2)$$

$$= \text{MAX}\{|x_1 - y_1|, |x_2 - y_2|\}$$

$$\le \text{MAX}\{\underbrace{|x_1 - z_1|}_{A} + \underbrace{|z_1 - y_1|}_{B}, \underbrace{|x_2 - z_2|}_{C} + \underbrace{|z_2 - y_2|}_{D}\}$$

$$\le \text{MAX}\{\text{MAX}\{A, C\} + \text{MAX}\{B, D\}, \text{MAX}\{A, C\} + \text{MAX}\{B, D\}\}$$

$$= \text{MAX}\{A, C\} + \text{MAX}\{B, D\}$$

$$= \underline{\quad\quad} \;\; \underline{\quad} - \underline{\quad\quad}$$

WANT ϕ S.T. $\nabla \phi = \langle y^2 \cos x, 2y \sin x + e^{2z}, 2y e^{2z} \rangle$

$$\phi_x = y^2 \cos x \implies \phi = y^2 \sin x + h(y, z)$$

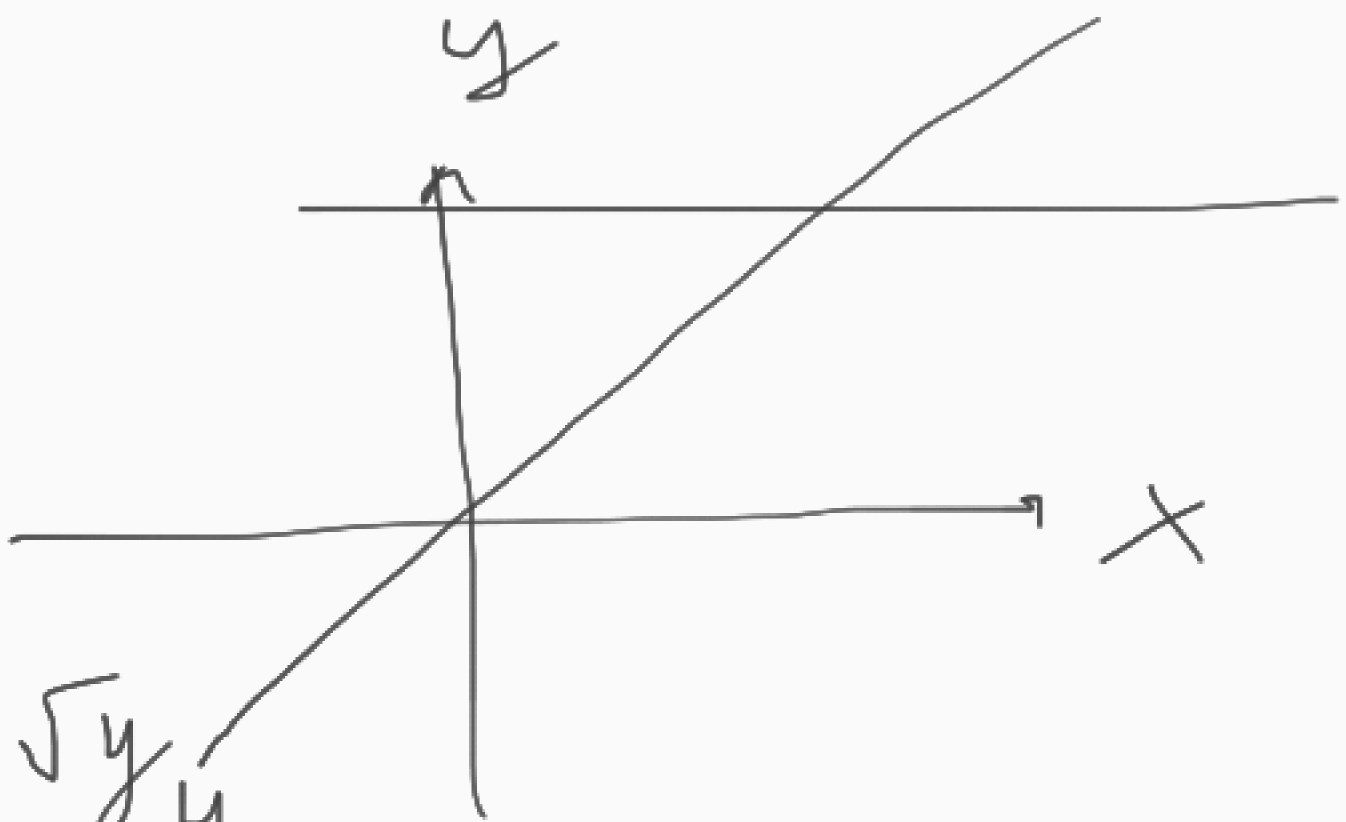
$$\implies \phi_y = 2y \sin x + h_y(y, z) = 2y \sin x + e^{2z}$$

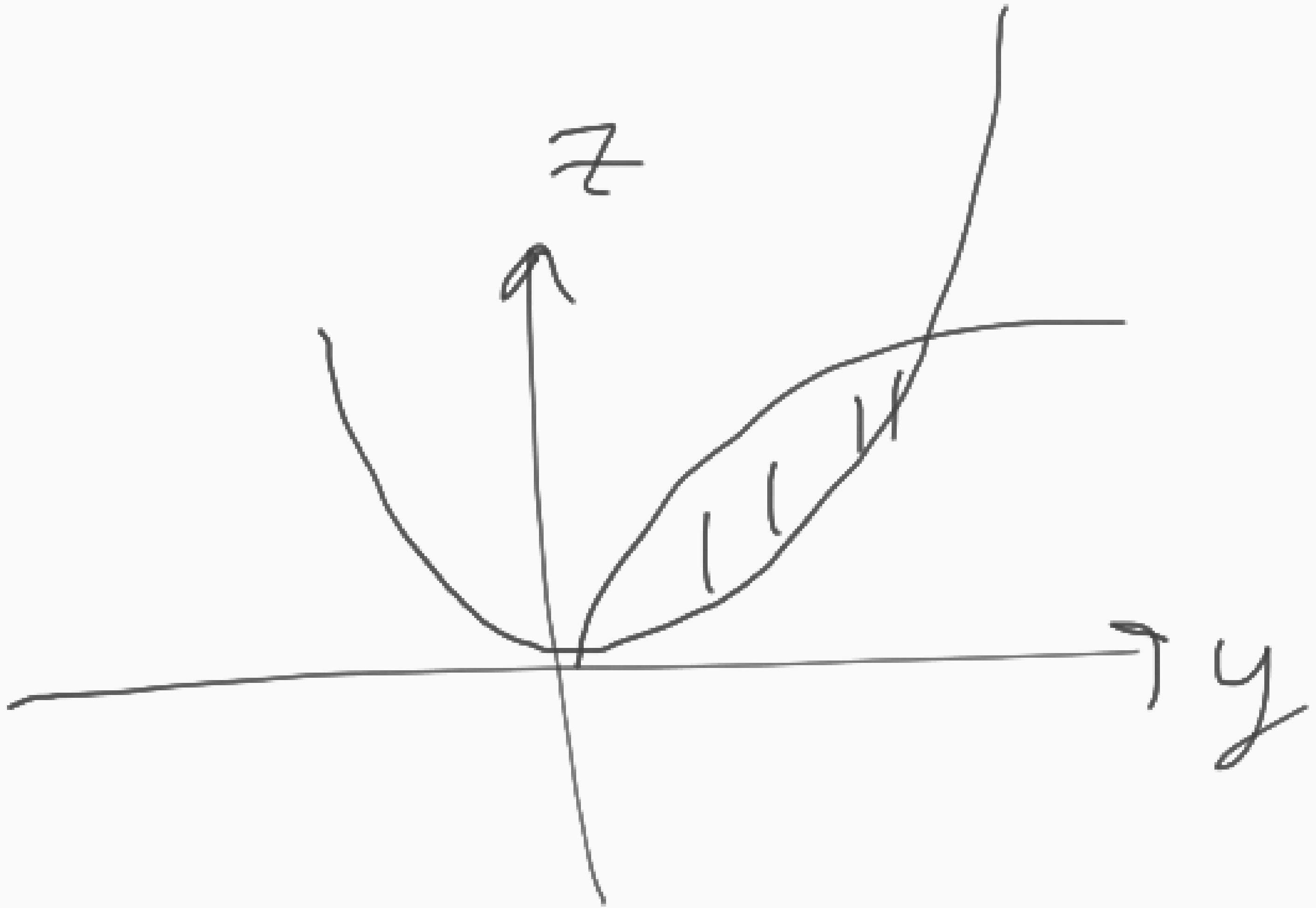
$$\implies h_y(y, z) = e^{2z} \implies h(y, z) = y e^{2z} + l(z)$$

$$\implies \phi_z = h_z(y, z) = 2y e^{2z} + l'(z) = 2y e^{2z}$$

$$\Rightarrow l'(z) = 0 \Rightarrow l(z) = C$$

$$\therefore \ \phi = y^2 \sin x + y e^{2z} + C$$





$$\int_0^1 \int_{y^2}^{\sqrt{y}} \int_0^{y} dx\, dz\, dy$$